Biost 517
Applied Biostatistics I

Scott S. Emerson, M.D., Ph.D.
Professor of Biostatistics
University of Washington

Lecture 3:
Overview of Descriptive Statistics

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Where am I going?

- Computing descriptive statistics is generally easy
  - 10th grade WASL
- Understanding what to use when and what they tell you is much harder
  - Important when it comes to inference:
    - "Parameters" are usually descriptive statistics on the population

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Lecture Outline

- Purpose of Descriptive Statistics
- General Methods
- Types of Measurements
- Types of Summary Measures
  - Univariate
  - Bivariate
  - Three or more variables

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Purpose of Descriptive Statistics

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Purpose of Descriptive Statistics

- Identify errors in measurement, data collection
- Characterize materials and methods
- Assess validity of assumptions needed for analysis
- Straightforward estimates to address scientific question
- Hypothesis generation

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Purpose: Identify errors

- Identify errors in measurement, data collection
  - Impossible, improbable, or inappropriate values
    - Univariate: Too low or too high
    - Multivariate: Strange combinations
  - Missing data
    - Univariate: Number missing by measurement
    - Multivariate: Predictors of missing data

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Purpose: Materials and Methods

- Characterize materials and methods
  - Describe subjects used in study
    - Univariate
      - Often broad ranges specified in inclusion/exclusion criteria
      - Want to know exact distributions obtained
    - Multivariately
      - Rarely are sample sizes defined for combinations of variables
      - E.g., in the sample are males old and females young?

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Purpose: Validity of Assumptions

- Assess validity of assumptions needed for analysis
  - Distributional assumptions
    - Within groups
      - e.g., exponential, Poisson distributions
    - Between groups
      - e.g., equal variances, proportional hazards
  - Modeling of dose response
    - Linearity of association
  - Influential or outlying cases

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Purpose: Confounding

- Assumptions about presence/absence of confounding
  - Confounding: A third variable confuses the estimation of an association between a predictor of interest and the outcome variable
  - Definition of a confounder:
    - Associated with the outcome (in a causal manner but not in pathway of interest)
    - Associated with the predictor of interest in the sample

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Example: Stress and Ulcers

- Alcohol consumption is thought to irritate stomach lining (thus causally associated with outcome)
- Many people drink alcohol when stressed (thus associated with predictor of interest)
  - If association truly exists in the population, it may well also exist in the sample
    - But consider randomization which (in some sense) precludes confounding

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Example: Stress and Ulcers

- Is EtOH consumption a confounder?
  - In causal pathway of interest?
    - Yes, if interested in all ways stress might cause ulcers
      - And if in causal pathway of interest, we would not want to adjust for EtOH as a confounder
    - No, if only interested in determining whether the physiologic consequences of stress cause ulcers
      - And if not in causal pathway of interest, then EtOH consumption would be confounding our ability to assess physiologic consequences

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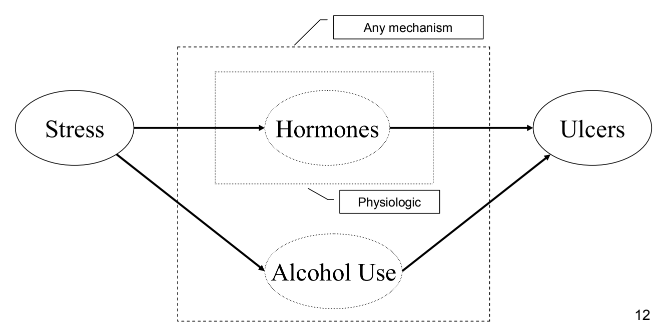
Example: Stress and Ulcers

- Causal pathway diagram



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Purpose: Preliminary Estimates

- Estimates for statistical inference
  - Many estimates used in statistical inference are based on sample descriptive statistics
  - E.g., method of moments estimators are defined by using sample moments (means, variances, etc.) to estimate population moments

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Purpose: Generate Hypotheses

- Exploring unanticipated effects
- Characterization of dose-response
  - Linear
  - U-shaped
  - Threshold
- Exploring difference in effects across subgroups
  - E.g., is association between treatment and clinical outcome similar in men and women

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General Methods

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General Process

- From study protocol
  - Describe sampling methods
  - Identify variables
    - Scientific role, statistical role, type of measurement
- From data
  - Univariate statistics
  - Bivariate statistics
  - Three or more variables

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1. Identify Sampling Scheme

- Describe sampling methods
  - Source of data
    - Location, time
    - Selection criteria
      - Inclusion criteria
      - Exclusion criteria

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Constrained Sample Sizes

- Sample sizes specified by design
    - Overall and/or within prespecified strata
    - E.g., cohort or case-control designs
- Sample sizes reflecting random process
    - Sometimes sampling scheme specifies time and location of sampling, not sample size
    - Allows estimate of prevalence or incidence
      - E.g., sample all cases of new lung cancer in Seattle during 1999
      - Sample size can be used to estimate incidence of lung cancer

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Common Study Designs

- Cross-sectional studies (surveys)
- Cohort studies
- Case-control studies
- Interventional studies

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Cross-sectional Studies

- Surveys of subjects sampled from a population
- Real or event time
- Efficient for examining
  - Common outcomes and risk factors
  - Associations (not cause and effect)
  - Can estimate prevalence of risk factors and outcomes
    - Overall and within groups

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Cohort Studies

- Groups defined by risk factor
  - Identified prospectively or retrospectively
- Followed longitudinally for outcome(s)
- Efficient for examining
  - Common outcomes
  - Many different outcomes for same exposure
  - Associations (not cause and effect)
  - Estimate incidence within risk factor groups
    - Cannot estimate prevalence of risk factor

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Case-Control Studies

- Groups defined by some outcome event
- Characterize prior exposures
  - Longitudinal study into the past
- Efficient for examining
  - Rare outcomes
  - Many different risk factors for same outcome
  - Associations (not cause and effect)
  - Estimate prevalence of exposure by disease
    - Cannot estimate prevalence of disease

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Interventional Studies

- Subjects assigned to some intervention
  - Ideally controlled, randomized
- Followed longitudinally for some outcome
  - So a special case of a cohort study
- Efficient for examining
  - Common outcomes
  - Cause and effect

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Conspiracies Against Laity I

- Random variables
  - A "random variable" is some measurement that might vary across subjects
    - Commonly denoted by capital letter or a mnemonic (e.g., *A* or *AGE* or *Age*)
  - We commonly denote the values a random variable can be using lower case letters
    - We talk about "events", e.g.,
      - $A = a_1$
      - $A \le a_5$
      - $a_{10} \le A \le a_{15}$

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Conspiracies Against Laity II

- Probability distributions
  - We know everything there is to know about a random variable when we can describe the probability of every possible event
  - Two common methods
    - Cumulative distribution function (cdf)
      - Know $Pr\ (Y \le y)$ for every possible value of y
    - Probability mass function (pmf)
      - Know $Pr\ (Y = y)$ for every possible value of y

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Conspiracies Against Laity III

- Conditional probability
  - Often we want to talk about the distribution of a random variable within a restricted group
    - E.g., the distribution of weight among males
  - Notation: *Wgt | Male*
    - Conditional CDF: $Pr\ (Wgt \le w \mid Male = m)$
      - *Conditional distn of weight among males*
        - » $Pr\ (Wgt \le w \mid Male = 1)$
      - *Conditional distn of weight among females*
        - » $Pr\ (Wgt \le w \mid Male = 0)$

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Conspiracies Against Laity IV

- Summary measures
  - We often summarize aspects of distributions
    - Mean (or expectation): $E\ (Y)$
    - Median: $Mdn\ (Y)$
    - Variance: $Var\ (Y)$
  - Summarizing conditional distributions
    - Conditional mean (or expectation): $E\ (Y \mid X{=}x\ )$
    - Median: $Mdn\ (Y \mid X = x)$
    - Variance: $Var\ (Y \mid X = x)$

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Detecting Associations

- Consider random variables
  - D be the disease state with values $(d_1, d_2, \ldots)$
  - R be a risk factor with values $(r_1, r_2, \ldots)$
- We consider the "statistical questions" that can be answered by study designs
  - Cross-sectional
  - Cohort
  - Case-control

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Detecting Associations

- Cross-sectional surveys show
  - $E(D \mid R = r_1) \neq E(D \mid R = r_2)$, OR
  - $E(R \mid D = d_1) \neq E(R \mid D = d_2)$
- Cohort studies show
  - $E(D \mid R = r_1) \neq E(D \mid R = r_2)$
- Case-control studies show
  - $E(R \mid D = d_1) \neq E(R \mid D = d_2)$

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Detecting Cause and Effect

- Demonstrated rigorously only through randomized studies
  - A characteristic of study design
  - There is nothing in the data that can distinguish between randomized studies and observational studies

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2. Identify Variables of Interest

- Identify variables of interest according to
  - Scientific meaning
  - Statistical role
  - Type of measurement

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Scientific Meaning of Variables

- Demographic variables
- Measures of exposure
- Measures of concurrent disease
- Measures of severity of disease
  - Cardiovascular function
  - Liver function
  - etc.
- Measures of clinical outcomes
- etc.

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Statistical Role of Variables

- Outcome (response) variable(s)
  - Primary and surrogates
- Predictor(s) of interest (define main groups)
- Subgroups of interest for effect modification
- Potential confounders
- Variables that add precision to analysis
  - Known to be associated with response
  - Often these are potential confounders
    - may be associated with predictor(s) of interest in sample
- Irrelevant to current question

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3. Identify Type of Measurement

- The way in which a variable is measured will affect the descriptive statistics that are of interest
  - Binary (dichotomous, Bernoulli)
  - Nominal (unordered categorical)
  - Ordered categorical
  - Quantitative
    - Discrete, interval continuous, ratio continuous
  - Censored

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Types of Measurements

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Characterizing Measurements

- Number of possible values
  - One, two, finite, countably/uncountably infinite
- Comparisons between values
  - Unordered, partially ordered, totally ordered
  - Scientific relevance of differences, ratios
- Completeness of measurement
  - Censoring

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Binary Measurements

- Only two possible values, which can be either
  - Labels, e.g., "Male" or "Female"
  - Coded as numbers, e.g., 1 or 2
- Most often it is statistically advantageous to represent as "indicator variables"
  - Possible values 0 or 1
  - 1 indicates the quality named by the variable
  - E.g., MALE is 1 for males, 0 for females
  - E.g., MARRIED is 1 for married, 0 for single, divorced, widowed, everything else

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Properties of Binary Measures

- Ordered
  - Differences (but not ratios) have a scientific interpretation
- The mean of an indicator variable is the proportion of subjects having the corresponding quality
  - Differences of means are scientifically relevant
  - Ratios of means are scientifically relevant
  - (Both differences and ratios of means may have limited ranges of interest for a specific problem)

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Categorical Measurements

- A finite number of possible values denoting qualities
  - E.g., occupation is laborer, clerical, professional, retired
  - E.g., marital status is single, cohabiting, married, divorced, separated, widowed
  - E.g., stage of cancer is I, II, III, or IV

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Unordered Categorical

- Unordered: no clear ordering of values can be prespecified
  - E.g., marital status
  - E.g., occupation status (unless used as a surrogate for physical exertion, sun exposure, etc.)

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Totally Ordered Categorical

- Totally ordered: categories can be qualitatively, but not quantitatively, ordered
  - Neither differences nor ratios have consistent scientific meaning
  - E.g., stage of cancer, degree of swelling

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Partially Ordered Categorical

- Partially ordered
  - Some categories have clear ordering, but others cannot be
  - E.g., Atypia on Pap smear often has "indeterminate" results
  - E.g., Severity of cancer might involve both grade and stage
    - May be hard to decide which is more severe:
      - » Low grade and high stage, or
      - » High grade and low stage

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Means of Categorical Variables

- Descriptively of less interest even for ordered
  - Spacing between categories is not well-defined
- However,
  - Means sometimes can still be used to identify (but not quantify) differences between distributions of categorical variables
  - Means may be particularly attractive in detecting shifts toward higher levels across groups with totally ordered categorical variables

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Quantitative Variables

- Values represent a (reasonably) precise quantification of some scientific measure
- Values can be
  - Discrete levels
    - No possible measurements between adjacent levels
    - E.g., counts of events
  - Continuous levels
    - E.g., weight
    - Distinction is often more a question of number of levels:
      - » Money is measured to nearest $0.01
      - » But often regarded as continuous

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Interval vs Ratio Measurements

- Generally, differences make sense for all quantitative variables
- Ratios only make sense if measurements are made relative to an absolute zero
  - Age, height, weight have absolute zeroes
  - Temperature has different zeroes in Farenheit and Celsius
- Categories of quantitative variables:
  - Interval: Only differences make clear sense
  - Ratio: Both differences and ratios of interest

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General Use of Ratios

- Ratios have have no scientific relevance with interval measurements
  - Thus not of great interest descriptively
  - May still be of use in identifying differences in distributions across groups
    - E.g., A ratio of temperatures different from 1 indicates different distributions
  - Quantifying differences in distributions will be specific to units used
    - Twice as hot in Farenheit vs twice as hot in Celsius

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Censored Variables

- A special type of missing data commonly arises in applications due to censored measurements (the exact value is not always known)
  - Right censoring: for some observations it is only known that the true value exceeds some threshold
  - Left censoring: for some observations it is only known that the true value is below some threshold
  - Interval censoring: for some observations it is only known that the true value is between two thresholds

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Example: Right Censoring

- Clinical trial detecting effect of aspirin on cardiovascular death
  - At the time of data analysis, death times have been observed for some subjects
  - At the time of data analysis, some subjects are still alive
- Representation of data using two variables
  - A variable measuring observation time until death or time of analysis, whichever comes first
  - An indicator variable telling which times are death times

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Types of Summary Measures

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Types of Summary Measures

- By feature of distribution
  - Typical value (location)
  - Spread of distribution (variability)
  - Symmetry of distribution (skewness)
  - Tendency to extreme values (kurtosis)
  - Depiction of entire distribution
- By number of variables described
  - Univariate
  - Bivariate
  - Higher dimensional

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Univariate Location

- Measures of location ("Typical value")
  - Numeric
    - Mode
    - Mean (arithmetic, geometric, harmonic)
    - Median (other percentiles)
    - Proportion exceeding a threshold
    - Odds of exceeding a threshold
  - Graphical
    - Mode of density

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Univariate Spread

- Measures of spread
  - Numeric
    - Range (min, max)
    - Interquartile range (25%ile, 75%ile)
    - Variance
    - Standard deviation
  - Graphical
    - Box plot
    - Histogram
    - Density

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Univariate Symmetry

- Measures of symmetry
  - Numeric
    - Coefficient of skewness
    - (Compare mean and median, etc.)
  - Graphical
    - Histogram
    - Density
    - Box plot

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Univariate "Heavy Tails"

- Measures of tendency to extreme values
  - Numeric
    - Coefficient of kurtosis

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Univariate Entire Distribution

- Numeric
  - Frequency tables
  - CDF tables
- Graphical
  - Histogram (stem-leaf)
  - Ogive
  - Density estimates
  - Empirical CDF, survival curves
  - Hazards
  - Box plots

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Univariate Descriptive Statistics

			Unordered	Ordered		
		Binary	Nominal	Categ	Quant	Cens
Entire Distribution	**Frequency**	OK	OK	OK	OK	
	Cum Freq	**boring**		OK	OK	KM
	Mode	**boring**	Sample	Sample	Density	
	Min / Max	**boring**		**boring**	OK	
Dicho-tomize	**Proportion (or Odds)**	OK	OK	OK	OK	KM
Quant-iles	**Quantiles (25^{th}, Mdn, 75^{th})**	**boring**		OK	OK	KM
Means	**Arithmetic**	(Prop)		***	OK	(?KM)
	Geometric				OK	(?KM)
	Harmonic				OK	(?KM)
	Std Dev	**boring**			OK	(?KM)
	Skew, Kurt	**boring**			OK	(?KM)

Bivariate Summary Measures

- Measures of association
  - Numeric
    - Stratified univariate descriptives
    - Slope of best fitting line
    - Correlation
    - Rank correlation
  - Graphical
    - Least squares line
    - Scatterplot smoother
    - Stratified box plots

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Bivariate Outliers

- Outliers: Data points far from any others
  - Numeric
    - Hat matrix
  - Graphical
    - Scatterplot

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Bivariate Entire Distribution

- Characterization of entire distribution
  - Numeric
    - Cross tabulation
  - Graphical
    - Scatterplot

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Three or More Variables

- Measures of association
  - Numeric
    - Stratified univariate descriptives
  - Graphical
    - Stratified least squares
    - Stratified scatterplot smoothers

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Three or More Variables

- Measures of interaction (effect modification)
  - Numeric
    - Stratified descriptives of bivariate association
  - Graphical
    - Stratified least squares
    - Stratified scatterplot smoothers

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Three or More Variables

- Measures of outlying values
  - Numeric
    - Hat matrix

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Three or More Variables

- Characterization of entire distribution
  - Numeric
    - Cross tabulation
  - Graphical
    - Stratified scatterplots

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What Do I Really Use?

- Univariate
  - Number of Missing
  - Mean
  - Standard Deviation
  - Min, Max
  - 25^{th}, 50^{th} (median), 75^{th} percentile
- Bivariate (and Trivariate)
  - Scatterplots (and smooths)
  - Stratified statistics

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