Biost 517
Applied Biostatistics I

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Lecture 16:
Two Sample Inference for
Correlated Response Data

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Lecture Outline

- Dependent Data Within Clusters
- Matched Continuous Data
  - Paired t Test (means, geometric means)
  - Sign Test (median difference)
  - (Wilcoxon) Signed Rank Test
- Comparing Proportions: Matched Samples

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Dependent Data Within Clusters

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Dependent Data

- There are times when data can not be presumed to be totally independent
  - Sampling within families
  - Sampling within schools, hospitals
  - Repeated measurements on individuals taken at a single time
  - Longitudinal data: repeated measurements taken on individuals over time

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Motivation for Longitudinal Data

- Three settings in which longitudinal studies are performed
  - Convenience of existing study population
  - Efficiency of using subjects as own comparison
  - Scientific questions about effects that occur
    - over time, or
    - within subjects

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Convenience

- Questions are truly cross-sectional
  - Multiple measurements made on each individual is easier than gathering new subjects
    - Natural variation within individuals provides additional information
  - E.g., Serum osmolality from Na, Glc, BUN
    - Interest is relationships between concurrent measurements

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Efficiency

- Questions could be answered with cross-sectional study
- Primary comparison within subjects may have less variability
  - Allow detection of smaller effects
  - E.g., Adjusting for baseline measurements
  - E.g., Cross-over study of a new treatment

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Example

- Percent glycosylated hemoglobin is used to monitor long term control in diabetes
  - Hemoglobin A1c
- Consider studies of two insulin delivery strategies
  - Independent groups
  - Cross-over design

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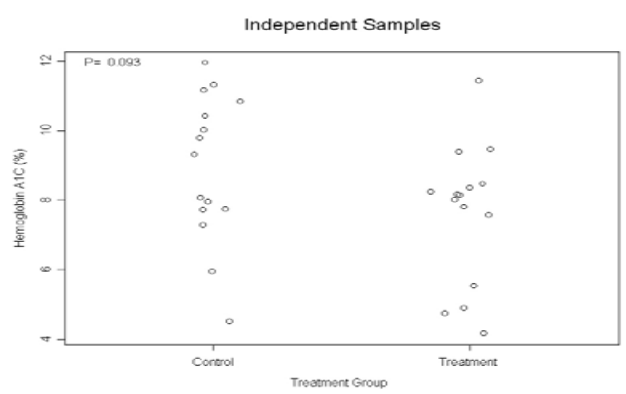
Graph: Independent Samples



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Inference: Independent Groups

- Large between-subject variability hampers our ability to detect differences
  - Between group SE is square root of sum of squared within group SEs
  - Within group SEs are proportional to within group standard deviation divided by the square root of n

$$se\left(\overline{X}-\overline{Y}\right)=\sqrt{\frac{\sigma_X^2}{n_X}+\frac{\sigma_Y^2}{n_Y}}$$

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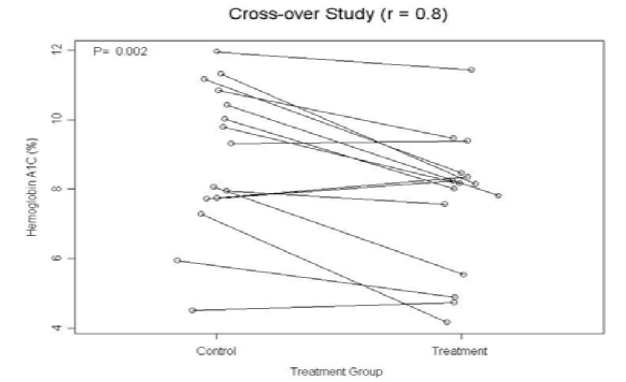
Graph: Cross-over Study



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Inference: Cross-over Study

- High correlation between measurements taken on the same individual increases precision
  - The "random effect" of patient ID can be thought of as a precision variable

$$se\left(\overline{X}-\overline{Y}\right)=se\left(\overline{D}\right)=\sqrt{\frac{\sigma_X^2+\sigma_Y^2-2\rho\sigma_X\sigma_Y}{n}}$$

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Longitudinal Questions

- Scientific questions about effects that occur over time
  - Studies to detect population time trends in response
    - E.g., rate (slope) of progression of retinopathy in population of diabetics over time
    - E.g., time to development of albuminuria

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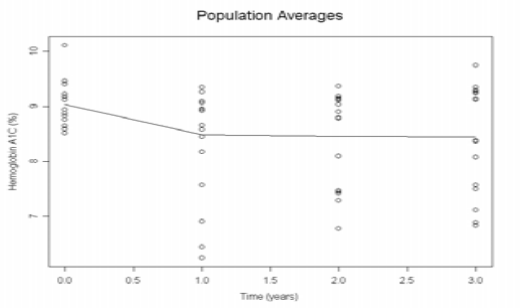
Example: "Marginal Effects"

- Time trends in group mean HbA1C
  - Note trends in mean and variability



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Within Subject Effects

- Trends in specific individuals might not look like trends in population means
  - Response over time may be restricted to subgroups of subjects
  - Response over time may be transient

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Longitudinal Scientific Questions

- Scientific questions about effects that occur within subjects
  - Studies to detect time trends or covariate effects in individual response
    - E.g., distribution of rates (slopes) of progression of retinopathy in population over time
    - E.g., effect of varying risk factors within individuals

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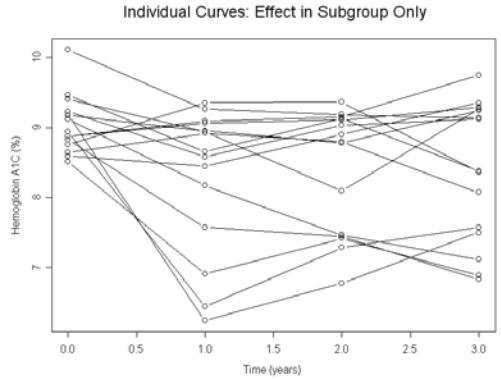
Effect in Subgroup



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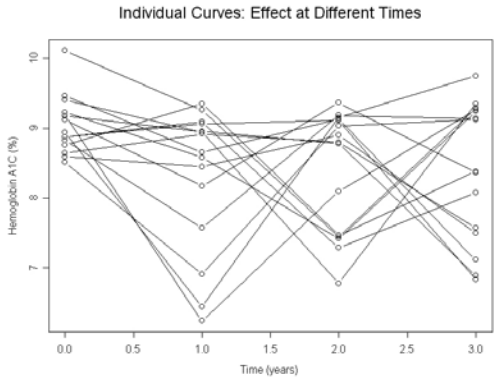
Transient Effects



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Choice of Measures of Outcome

- In order of importance
  - Scientific relevance
    - Including state of current knowledge
  - Plausibility of difference across groups
  - Statistical precision for analysis

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Longitudinal Outcome Measures

- In longitudinal studies, each individual may have multiple measurements over time
  - Definition of individual response thus can be based on multiple measurements
    - Response at a fixed time
    - Responses at multiple fixed times
    - Average response over time (area under curve)
    - Rate of change in response (slope)
    - Time to attaining some level of response

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Measures of Outcome

- "Marginal" or population effects
  - Difference or ratio of group means, geometric means, medians, proportion or odds above threshold, hazards
  - Pr (Y > X)
- "Within subject" effects
  - Mean, median difference
  - Mean, geometric mean, median ratio
  - Within subject odds ratio
  - Pr (Y > X)

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Choice of Longitudinal Outcome

- Should reflect scientific relevance, plausibility of effect, precision
  - Final level of response may be more important than earlier effects
    - (But in the long run, we are all dead)
  - Summarizing response at multiple time points reflects population rather than individuals
  - Average response over time sensitive to transient effects
  - Differences in time to event may be clinically meaningless

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Statistical Issues

- Repeated measurements on subjects require special analysis techniques
  - May have erroneous conclusion if fail to account for correlated observations
    - Point estimates may be biased for population parameters
      - Too much emphasis placed on some subjects
    - Confidence intervals will not be accurate representation of our true confidence
    - P values will be wrong

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Statistical Approaches

- Three basic approaches to analyzing correlated data
  - Reduce measurements on each cluster to a single observation; analyze across clusters
  - Estimate correlation within clusters and adjust standard errors for population based models
    - GEE, marginal models
    - "Robust" variance estimates
  - Adjust estimates for "random effects"
    - "Mixed effects models": both fixed and random

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Easiest Approach

- Reduce data for each individual to a single measurement
  - E.g., response at end of study, average response, rate of change
  - Analyses can then be based on standard methods for independent data
  - But:
    - Does not allow time-varying covariates
    - May not be most efficient statistically

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Example: Beta-carotene Data

- Randomized clinical trial of beta-carotene supplementation on plasma levels of beta-carotene and vitamin E
  - Subjects randomized to 5 dose groups
  - Measurements at baseline, after 3 and 9 months of treatment, and 3 months after stopping treatment
  - Scientific question: How do plasma beta-carotene levels change over time within dose groups?
    - (effect modification between dose and time)

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Example: Beta-carotene Data

- Reduce data to a single measurement on each subject
  - Difference between follow-up and baseline
    - Consider average of differences
    - No change corresponds to a difference of 0
  - Ratio between follow-up and baseline
    - Consider average of ratios
    - No change corresponds to a ratio of 1

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Example: SEP data

- Somatosensory evoked potential measurements on healthy adults
  - Measurements of nerve conduction time
    - Four separate peaks for each leg of each subject
  - Reduce data to a single measurement
    - Consider only one peak on one leg
      - Which one?
    - Average measurements across peaks, legs
      - But will only generalize to similar averages
    - (Differences between peaks?)

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Matched Continuous Data

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Comparing Means

- Paired t test
  - Compute differences for each pair
  - One sample t test that mean difference is 0
- Note that mean difference is difference of means
  - Same answer for population (“marginal”) and within subject questions (providing they both make sense)
    - May be inherent confounding, effect modification
    - E.g., age vs time vs birth year cohort effects

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Comparing Geometric Means

- Paired t test on log transformed data
  - Compute differences for each pair
  - One sample t test that mean difference is 0
  - Back transform to consider geometric mean of ratios
    - Also geometric mean of ratios

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Sign Test

- A very simple alternative test to the paired t test (which compares means) is to test whether the median of the differences is zero
  - If the median of the differences is zero, we would expect as many differences to be above zero as below zero
    - The differences that are exactly zero do not contribute much information about which measurement tends to be higher

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Median Difference

- Compute differences of observations
  - Consider whether differences tend to be negative or positive

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Median Difference Properties

- Median difference is not difference in medians
  - Ex: X = (1, 3, 10); Y = (2, 5, 10)
    - mdn(Y) – mdn(X) = 5 – 3 = 2
    - Difference: D = X – Y = (1, 2, 0); mdn(D) = 1
- The median difference is not transitive
  - Ex: X = (1, 2, 3); Y= (2, 3, 1); Z = (3, 0, 2)
    - mdn (Y – X) = 1 > 0 (so "Y larger than X")
    - mdn (Z – Y) = 1 > 0 (so "Z larger than Y")
    - mdn (X – Z) = 1 > 0 (so "X larger than Z")

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Sign Test (Elevator Statistics)

- Proportion positive among nonzero differences

$$X_i \overset{iid}{\sim} (\mu, \sigma^2) \quad Y_i \overset{iid}{\sim} (\nu, \tau^2) \quad D_i = X_i - Y_i \overset{iid}{\sim} (\mu - \nu, \omega^2)$$

$$P = \text{number of } D_i\text{'s} > 0$$

$$N = \text{number of } D_i\text{'s} < 0$$

If the median difference is 0, the number of positive differences is binomially distributed:

$$H_0 : P \sim B(P + N, 0.5)$$

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Sign Test: Stata Commands

- Stata has a command to perform the sign test
  - "`signtest var1 = var2`"
    - Provides one-sided and two-sided P values
    - Does not provide any meaningful estimates or confidence intervals
  - (The sign test can also be performed by creating the differences, changing the zeroes to missing, and then using "`bitest`")

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Sign Test: Stata Example

- Example: Change in plasma beta-carotene in placebo group

```
. signtest carot3=carot0 if dose==0
Sign test
    sign |    observed    expected
---------+------------------------
positive |           1         3.5
negative |           6         3.5
    zero |           0           0
---------+------------------------
     all |           7           7
```

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Sign Test: Stata Example

```
One-sided tests:
Ho: mdnn of carot3 - carot0  = 0 vs.
Ha: median of carot3 - carot0  > 0
Pr(#pos >= 1) = Binomial(n=7, x>=1, p=0.5)=  0.9922

Ho: median of carot3 - carot0  = 0 vs.
Ha: median of carot3 - carot0  < 0
Pr(#neg >= 6) = Binomial(n=7, x>=6, p=0.5)=  0.0625

Two-sided test:
Ho: median of carot3 - carot0  = 0 vs.
Ha: median of carot3 - carot0  ~= 0
Pr(#pos >= 6 or #neg >= 6)               =  0.1250
```

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Interpretation

- We can not with 95% confidence reject the null hypothesis that the median change in plasma beta-carotene levels after 9 months of treatment with placebo was 0

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(Wilcoxon) Signed Rank Test

- The sign test is simple to perform, but it ignores a lot of information
  - Intuitively, you would expect that there is some information in the magnitude of the differences as well as the sign
    - For instance, there may be nearly as many negative differences as positive differences, but the positive differences tend to be far larger (in absolute value) than the negative differences

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(Wilcoxon) Signed Rank Test

- The Wilcoxon signed rank test attempts to use the information about the magnitude of the differences
  - The null hypothesis of the Wilcoxon signed rank test is that
    - the number of positive and negative differences should tend to be equal, and
    - there should be no tendency for the positive differences to be further from (or closer to) zero than the negative differences

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(Wilcoxon) Signed Rank Test

- Basic approach of the signed rank test
  - Compute the differences and rank the absolute value of the differences
  - Sum up the ranks of the positive differences
  - Under the null hypothesis of equality of distributions, the sampling distribution for that sum should be the same as randomly choosing n/2 numbers from the integers 1 to n
    - Adjustment for ties and zeroes
    - (Computers can figure this out for us)

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Example of Signed Ranks

X	$\{9,\ 7,\ 4,\ 2,\ 37,\ 9,\ 7,\ 4\}$
Y	$\{3,\ 8,\ 4,\ 5,\ 7,\ 5,\ 9,\ 5\}$
Diff	$\{6, -1,\ 0, -3,\ 30,\ 4, -2, -1\}$
Ranks	$\{7, 2.5,\ 1,\ 5,\ 8,\ 6,\ 4, 2.5\}$

Sum of Positive Ranks : 21

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Summary Measure

- It is not immediately clear (or easily explained) what aspect of the distributions the signed rank test is comparing
  - Can be significant because
    - Number of positive differences is unusually high
    - Mean positive difference is high
  - It provides some sort of a balance between the two

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Interpretation

- In any case, it is clear that a significant signed rank test can only be interpreted as a difference in distributions
  - The standard error of the test statistic is based on a permutation distribution, and thus
    - is only testing equality of distributions with the appropriate type I error,
    - but because it is not a consistent test of arbitrary differences between distributions
      - the differences must be something that the signed rank test can detect

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Stata Commands

- Stata has a command to perform the signed rank test
  - "`signrank var1 = var2`"
    - Provides one-sided and two-sided P values
    - Does not provide any meaningful estimates or confidence intervals

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Stata Example

- Example: Change in plasma beta-carotene in placebo group

```
. signrank carot3=carot0 if dose==0
Wilcoxon signed-rank test
    sign |      obs   sum ranks    expected
---------+--------------------------------
positive |        1           1          14
negative |        6          27          14
    zero |        0           0           0
---------+--------------------------------
     all |        7          28          28
(some purely technical output omitted)
Ho: carot3 = carot0    z =  -2.197 Prob > |z| =   0.0280
```

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Interpretation

- We can with 95% confidence reject the null hypothesis that there was no systematic trend toward increasing or decreasing plasma beta-carotene levels after 9 months of treatment with placebo
  - (Note that we were able to reject the null with the signed rank, but not the sign test.)

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Comparing Proportions: Matched Samples

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Matched Binary Data

- In some studies, we make comparisons of proportions across samples which are not independent
  - E.g., Cross-over studies
    - Relief of headaches from aspirin vs Tylenol
    - Each subject receives each treatment (in random order)
  - E.g., Ophthalmology studies
    - Cure of conjunctivitis: new treatment vs placebo
    - Each subject receives each treatment (randomize which eye receives new treatment)

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Presentation of Data

- We tend to alter the format of contingency table to reflect the matched data
  - Instead of response by group, we display concordance of response in each group

Response

Treatment	+	−	
New	r	s	n
Plc	t	u	n
	m_0	m_1	n

Resp on Plc

Resp on New	+	−	
+	a	b	r
−	c	d	s
	t	u	n

Estimate

- Usual estimate of difference of proportions

Resp on Plc

Resp on New	+	−	
+	a	b	r
−	c	d	s
	t	u	n

Estimated difference in proportions

$$\frac{r}{n} - \frac{t}{n} = \frac{b-c}{n}$$

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Analysis of Data

- The analysis of the matched data can proceed along two lines
  - Least frequently used
    - Compare proportion with response in each group taking matching into account
    - Analogous to paired t test (which would be a valid test in large samples)
  - Most often used: McNemar's test
    - Focus on the "discordant pairs" only
    - Evaluate whether discordant pairs are evenly distributed between (+, -) and (-, +)

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McNemar's Test: Rationale

- If response were equal in the two groups, discordant pairs should be equally likely to be in either order
  - Condition on the number of discordant pairs
    - Intuitively, the number of discordant pairs does not contribute much information as to which group does better
  - Under the null hypothesis, the discordant pairs should be equally likely to be in either the "b" or the "c" cell of the contingency table
    - Use the one sample test of a binomial proportion

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McNemar's Test

- One sample binomial test

		Resp on Plc +	Resp on Plc −	
Resp on New	+	a	b	r
	−	c	d	s
		t	u	n

If response rates are equal for both treatments, under the null we would have binomial distribution

$$b \sim B(b+c, 0.5)$$

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Stata: Exact McNemar's

- Example: Prevalence of edema vs ascites in liver data
  - Are ascites and edema equally prevalent?
    - Stata does not perform McNemar's using exact distributions, but we can get it to perform the test quite easily

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Stata: Exact McNemar's

```
table edema ascites
----------+-----------
          |  ascites
    edema |    0     1
----------+-----------
        0 |  268     7
        1 |   20    17

----------+-----------
```

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Stata: Exact McNemar's

```
. bitesti 27 7 0.5
N   Obs k   Exp k   Assumed p   Observed p
----------------------------------------
27      7    13.5    0.50000      0.25926

Pr(k>= 7)           = 0.9970     (one-sided test)
Pr(k<= 7)           = 0.0096     (one-sided test)
Pr(k<= 7 or k>= 20) = 0.0192     (two-sided)
```

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McNemar's Test

- Test statistic can be based on asymptotic distribution
  - Standardized Z statistic or (more commonly) a chi squared statistic

Under the null we would have binomial distribution

$$b \sim B(b+c, 0.5)$$

$$Z = \frac{\frac{b}{b+c} - 0.5}{\sqrt{0.25/(b+c)}} \overset{H_0}{\sim} N(0,1) \qquad \chi^2 = Z^2 = \frac{(b\text{-}c)^2}{(b+c)}$$

Stata: Large Sample

- Stata uses asymptotic theory
  - "mcc casevar ctrlvar"
    - mcc = matched case-control
    - Labels are by "Cases" and "Controls"
    - Provides two-sided P-values
    - Provides confidence interval for difference in proportions

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Stata Commands: Example

- Prevalence of edema vs ascites in liver data

```
mcc edema ascites
                Controls
Cases     |Exposed  Unexposed  |   Total
----------+--------------------+---------
  Exposed |     17         20  |      37
Unexposed |      7        268  |     275
----------+--------------------+---------
    Total |     24        288  |     312
McNemar's chi2(1)=  6.26  Pr>chi2= 0.0124
```

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Stata Commands: Example

- Prevalence of edema vs ascites in liver data

```
Proportion with factor
Cases       .1186
Controls    .0770     [95% CI]
---------     --------------------
difference  .0417   .0061   .0772
ratio      1.5467  1.0954  2.1698
rel. diff.  .0451   .0106   .0797

odds ratio 2.8571  1.1605  7.9971 (exact)
```

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Compare Paired t Test

```
ttest edema=ascites
Paired t test           Number of obs =      312
------------------------------------------------
Variable | Mean  St Err  t     P>|t|   [95% CI]
---------+--------------------------------------
   edema | .1186  .0183 6.469  0.0000 .0825  .1547
 ascites | .0769  .0151 5.091  0.0000 .0472  .1067
---------+--------------------------------------
    diff | .0417  .0165 2.523  0.0121 .0092  .0742
```

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Compare Paired t Test

```
Degrees of freedom: 311

             Ho: mean diff = 0

Ha: diff < 0    Ha: diff ~= 0      Ha: diff > 0
t =  2.523      t =  2.523         t =  2.523
P < t = 0.9939  P > |t| = 0.0121   P > t = 0.0061
```

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Comments

- It is useful to highlight the difference between the questions answered by the chi square test and McNemar's test
  - Consider test of edema and ascites
    - McNemar's test
      - Are ascites and edema equally prevalent?
    - Chi square test
      - Does the prevalence of ascites differ between subjects with and without edema?

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Sign Test vs McNemar's Test

- McNemar's test is just the sign test performed on binary data
  - The sign test is a more general description of the procedure, and thus I prefer using that name even when using binary data
  - Hence, I introduced the word "McNemar" only because you will sometimes see it referred to in the literature
    - I wish the word "McNemar" would disappear from the literature (my brain is full)

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