**Biost 518: Applied Biostatistics II**

**Biost 515: Biostatistics II**

Emerson, Winter 2014

**Homework #8**

February 28, 2014

**Written problems:** To be submitted as a MS-Word compatible file to the class Catalyst dropbox by 9:30 am on Friday, March 7, 2014. See the instructions for peer grading of the homework that are posted on the web pages.

*On this (as all homeworks) Stata / R code and unedited Stata / R output is* ***TOTALLY*** *unacceptable. Instead, prepare a table of statistics gleaned from the Stata output. The table should be appropriate for inclusion in a scientific report, with all statistics rounded to a reasonable number of significant digits. (I am interested in how statistics are used to answer the scientific question.)*

***Unless explicitly told otherwise in the statement of the problem, in all problems requesting “statistical analyses” (either descriptive or inferential), you should present both***

* ***Methods: A brief sentence or paragraph describing the statistical methods you used. This should be using wording suitable for a scientific journal, though it might be a little more detailed. A reader should be able to reproduce your analysis. DO NOT PROVIDE Stata OR R CODE.***
* ***Inference: A paragraph providing full statistical inference in answer to the question. Please see the supplementary document relating to “Reporting Associations” for details.***

All problems refer to the salary dataset as found on the class web pages. This is a very large file, so you need to make sure you have sufficient memory available when you start Stata. Also, it is probably most convenient if you code the variables as numbers, and use labels to make them more understandable. The following file on the Datasets web pages contains commands you might find useful.

http://www.emersonstatistics.com/datasets/initsalary.doc

1. We are interested in making inference about the difference in the mean monthly salary paid to women faculty in 1995 and that paid to men faculty in 1995. In this problem, we focus on alternative modeling of the variables *yrdeg* and *startyr*. In all models in this problem, we will appropriately adjust for degree, field, administrative duties, and sex. ***(Note that I have provided answers to all parts of this problem except parts a, b and i, which you should answer.)***
   1. In all parts of this problem, in addition to the year of degree and year starting at the UW, you should adjust for the highest degree obtained, field, and administrative duties. What is the best way to model the variables *degree, field,* and *admin*? Briefly justify your answer.
   2. In all parts of this problem you should use robust standard error estimates. Briefly explain why inference based on classical linear regression (without robust SE estimates) would be incorrect. Do you think the classical linear regression inference would tend to be conservative or anti-conservative? Justify your answer.
   3. Model *yrdeg* and *startyr* as linear continuous variables. Report the inference you would make for the difference in mean salaries for men and women (a table of the results for parts c, d, e, f, and g will be sufficient).

**Ans: (See table below)**

* 1. Model *yrdeg* and *startyr* as quadratic continuous variables (so linear continuous plus a second order term). Report the inference you would make for the difference in mean salaries for men and women (a table of the results for parts c, d, e, f, and g will be sufficient).

**Ans: (See table below)**

* 1. Model *yrdeg* and *startyr* as dummy variables for groups defined by earlier than 1960, 1960-64, 1965-69, 1970-74, 1975-79, 1980-84, 1985-89, and 1990 or later. Report the inference you would make for the difference in mean salaries for men and women (a table of the results for parts c, d, e, f, and g will be sufficient).

**Ans: (See table below)**

* 1. Model *yrdeg* and *startyr* as linear splines with knots at years 1960, 1965, 1970, 1975, 1980, 1985, and 1990. Report the inference you would make for the difference in mean salaries for men and women (a t able of the results for parts c, d, e, f, and g will be sufficient).

**Ans: (See table below)**

* 1. Repeat parts c – f when modeling the ratio of mean salaries across sexes and when modeling the ratio of geometric mean salaries across sexes. These results can be included in the same table.)

**Ans: (See table below)**

* 1. Examine the agreement between the inference about the adjusted association between monthly salary and sex. Did the inference vary substantially across the various models?

**Ans: The following table provides the regression parameter estimates for the predictor indicating female sex, its Z statistic, its two-sided P value, and its 95% CI for the alternative methods of modeling year of degree and starting year. A few comments are in order**

* **In all cases, the linear splines provided the best fit to the data in the sense that adding the linear splines to each of the other models proved to be statistically significant. Adding the dummy variables to the model that included the linear splines did not improve the fit. I do not recommend doing this sort of testing unless your question was about the form of the relationship (e.g., linear vs nonlinear). My point here is that the linear splines did seem to model the true relationship with salary better when I was modeling sex, field, degree, and administrative duties.**
* **When modeling year of degree and start year as quadratic functions, I could not statistically establish nonlinearity in the linear regression model of the difference of means. When considering ratios of means or geometric means, I could detect the nonlinearity of either the year of degree or starting year when testing them combined, but because the terms are so correlated, I could not ensure that both were nonlinear when adjusting for the other.**
* **When modeling year of degree and start year as dummy variables or linear splines, there tended to be statistically significant departures from linearity for each variable separately and combined.**
* **Note that I included the Z statistic in this table only because the results were so strikingly statistically significant, that is only through looking at the Z statistic that we can assess whether there were any substantial differences (there were not).**
* **Note the similarity in ratios across all methods of modeling year of degree and start years and across the summary measures (means or geometric means).**
* **I provided inference about ratios of means using both Poisson regression and the generalized linear model when assuming Gaussian data with a log link. I prefer the Poisson regression, though this really only makes a big difference when looking at risk ratios with binary data. In that case, I *highly* recommend using Poisson regression rather than the generalized linear model with the binomial family and the log link. With means of positive continous random variables Poisson regression or the Gaussian GLM will both tend to behave okay.**
* **Lastly, the difference in means is of course a very different scale than the ratios of means or geometric means. But if you consider that the mean monthly salary for the entire sample was $6,389.81, the difference in means of about $420 is about 7% of the overall mean. So all models are giving quite similar answers.**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Estimate** | **Z** | **P Value** | **95% CI low** | **95% CI high** |
| *Difference in Means* | | | | | |
| **Linear** | -428.3 | -5.23 | < .0001 | -588.9 | -267.8 |
| **Quadratic** | -428.1 | -5.25 | < .0001 | -588.1 | -268.0 |
| **Dummy** | -447.7 | -5.45 | < .0001 | -609.0 | -286.5 |
| **Splines** | -419.7 | -5.17 | < .0001 | -579.0 | -260.5 |
| *Ratio of Means (Poisson)* | | | | | |
| **Linear** | 0.9266 | -5.42 | < .0001 | 0.9014 | 0.9525 |
| **Quadratic** | 0.9280 | -5.36 | < .0001 | 0.9030 | 0.9537 |
| **Dummy** | 0.9244 | -5.63 | < .0001 | 0.8994 | 0.9500 |
| **Splines** | 0.9289 | -5.34 | < .0001 | 0.9041 | 0.9544 |
| *Ratio of Means (GLM)* | | | | | |
| **Linear** | 0.9227 | -5.55 | < .0001 | 0.8969 | 0.9493 |
| **Quadratic** | 0.9246 | -5.43 | < .0001 | 0.8988 | 0.9511 |
| **Dummy** | 0.9185 | -5.83 | < .0001 | 0.8926 | 0.9451 |
| **Splines** | 0.9245 | -5.49 | < .0001 | 0.8989 | 0.9508 |
| *Ratio of Geometric Means* | | | | | |
| **Linear** | 0.9347 | -5.22 | < .0001 | 0.9113 | 0.9587 |
| **Quadratic** | 0.9352 | -5.22 | < .0001 | 0.9119 | 0.9590 |
| **Dummy** | 0.9328 | -5.42 | < .0001 | 0.9096 | 0.9566 |
| **Splines** | 0.9363 | -5.17 | < .0001 | 0.9132 | 0.9600 |

* 1. In a real situation, how would choose among the alternative methods for adjusting for year of degree and starting year?

1. We are interested in making inference about the difference in the mean monthly salary paid to faculty according to the year in which faculty obtained their degree and the year in which they started at UW. In all models in this problem, we will appropriately adjust for degree, field, administrative duties, and sex.
   1. Provide inference about the adjusted association between monthly salary and year of degree (modeled as a linear continuous variable, not adjusted for starting year). **(SEE TABLE BELOW (D))**
   2. Provide inference about the adjusted association between monthly salary and starting year (modeled as a linear continuous variable, not adjusted for year of degree). **(SEE TABLE BELOW (D))**
   3. Provide inference about the adjusted association between monthly salary and year of degree (modeled as a linear continuous variable, and adjusted for starting year as well as the other variables). **(SEE TABLE BELOW (D))**
   4. Provide inference about the adjusted association between monthly salary and starting year (modeled as a linear continuous variable, and adjusted for year of degree as well as the other variables). **(SEE TABLE BELOW)**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | **Individual** | | | **Adjusted** | | |
| **Variable** | **Estimate** | **95% CI** | **P val** | **Estimate** | **95% CI** | **P val** |
| **Year of Degree** | -19.14 | -22.23, -16.06 | .000 | -88.49 | -94.64, -82.34 | .000 |
| **Starting Year** | 16.73 | 13.62, 19.84 | .000 | 81.59 | 75.60, 87.57 | .000 |

The univariate analysis in parts a and b are easy enough to interpret on their own. First, we predict salary considering the year highest degree was granted. Then salary as associated with starting year at UW. Parts c and d take into account experience prior to employment at UW .

Problems 3 – 5 ask you to fit a series of models in which you consider a hierarchy of adjusted analyses for each of three different summary measures. Your response to these problems might be best presented in a table of inference about the adjusted association between monthly salary and sex.

For the benefit of the graders, we will agree on modeling *yrdeg* and *startyr* as linear splines as computed in problem 1f.

1. We are interested in making inference about the difference in the mean monthly salary paid to women faculty in 1995 and that paid to men faculty in 1995.
   1. Report inference regarding the unadjusted comparison of women’s and men’s salaries.
   2. Report inference regarding the comparison of women’s and men’s salaries after adjustment for degree.
   3. Report inference regarding the comparison of women’s and men’s salaries after adjustment for degree, year of degree.
   4. Report inference regarding the comparison of women’s and men’s salaries after adjustment for degree, year of degree, starting year at UW.
   5. Report inference regarding the comparison of women’s and men’s salaries after adjustment for degree, year of degree, starting year at UW, field.
   6. Report inference regarding the comparison of women’s and men’s salaries after adjustment for degree, year of degree, starting year at UW, field, administrative duties. Save the predicted values of the mean salary for each individual as *fit3.*
   7. Report inference regarding the comparison of women’s and men’s salaries after adjustment for degree, year of degree, starting year at UW, field, administrative duties, rank.

|  |  |  |  |
| --- | --- | --- | --- |
| **Salary~** | **Mean Diff.** | **95% CI** | **P val** |
| **Male** | 1334.73 | 1148.29, 1521.18 | .000 |
| **Male+Degree** | 1266.16 | -1418.03, -586.70 | .000 |
| **Male+Degree+YrDeg** | 614.13 | 446.02, 782.24 | .000 |
| **Male+Degree+YrDeg+StartYr** | 614.58 | 443.85, 785.31 | .000 |
| **Male+Degree+YrDeg+StartYr+Field** | 420.05 | 256.99, 583.12 | .000 |
| **Male+Degree+YrDeg+StartYr+Field+Admin** | 419.73 | 260.47, 578.99 | .000 |
| **Male+Degree+YrDeg+StartYr+Field+Admin+Rank** | 280.66 | 145.81, 415.52 | .000 |

Using linear regression, we find that as we adjust for possible confounders, the mean difference between male and female salary moves towards the null hypothesis, although it fails to reach a reasonable threshold for equality. After adjusting for all variables, mean salary for males is $280.66 greater than female mean salary. Based on a 95% confidence interval, it would not be unusal if the true mean difference were between $145.81 and $415.52 with males tending to greater mean salary than females. At the 5% significance level, we reject the null hypothesis of equal mean salary (P<0.001).

1. We are interested in making inference about the ratio of geometric mean monthly salary paid to women faculty in 1995 and that paid to men faculty in 1995.
   1. Report inference regarding the unadjusted comparison of women’s and men’s salaries.
   2. Report inference regarding the comparison of women’s and men’s salaries after adjustment for degree.
   3. Report inference regarding the comparison of women’s and men’s salaries after adjustment for degree, year of degree.
   4. Report inference regarding the comparison of women’s and men’s salaries after adjustment for degree, year of degree, starting year at UW.
   5. Report inference regarding the comparison of women’s and men’s salaries after adjustment for degree, year of degree, starting year at UW, field.
   6. Report inference regarding the comparison of women’s and men’s salaries after adjustment for degree, year of degree, starting year at UW, field, administrative duties. Save the predicted values of the geometric mean salary for each individual as *fit4.*
   7. Report inference regarding the comparison of women’s and men’s salaries after adjustment for degree, year of degree, starting year at UW, field, administrative duties, rank.

|  |  |  |  |
| --- | --- | --- | --- |
| **Salary~** | **Ratio of Geometric Means** | **95% CI** | **P Value** |
| **Male** | 1.232 | 1.195, 1.269 | <.001 |
| **Male+Degree** | 1.219 | 1.183, 1.256 | <.001 |
| **Male+Degree+YrDeg** | 1.100 | 1.071, 1.130 | <.001 |
| **Male+Degree+YrDeg+StartYr** | 1.101 | 1.071, .119 | <.001 |
| **Male+Degree+YrDeg+StartYr+Field** | 1.068 | 1.041, 1.096 | <.001 |
| **Male+Degree+YrDeg+StartYr+Field+Admin** | 1.068 | 1.042, 1.095 | <.001 |
| **Male+Degree+YrDeg+StartYr+Field+Admin+Rank** | 1.046 | 1.023, 1.067 | <.001 |

Using linear regression that considers the log transformation of salaries, we find that as we adjust for possible confounders, the ratio of geometric means comparing male and female salary moves towards the null hypothesis, although it fails to reach a reasonable threshold for equality. After adjusting for all variables, geometric mean salary for males is 4.6% greater than female geometric mean salary. Based on a 95% confidence interval, it would not be unusual if the true geometric mean for males were anywhere between 2.3% and 6.7% greater than for females. At the 5% significance level, we reject the null hypothesis of equal mean salary (P<0.001).

1. We are interested in making inference about the ratio of the mean monthly salary paid to women faculty in 1995 and that paid to men faculty in 1995. You can use Poisson regression (with the irr option to get exponentiated parameter estimates), or you can use a generalized linear model with a log link. Stata has a regression function “glm” that allows the specification of a log link function. Hence, you can fit the regression for part a using the command

glm salary female if year==95, link(log) robust

Parameter estimates will be interpretable as the log mean (intercept) and log mean ratio (slope). (glm stands for “generalized linear model” and it includes as special cases linear regression, logistic regression, and Poisson regression. By default, it presumes the data are continuous and models the mean according to the value of the link function.) By specifying the “eform” option, it will return the exponentiated parameter estimates.

In either case, make clear which analysis method you used.

* 1. Report inference regarding the unadjusted comparison of women’s and men’s salaries.
  2. Report inference regarding the comparison of women’s and men’s salaries after adjustment for degree.
  3. Report inference regarding the comparison of women’s and men’s salaries after adjustment for degree, year of degree.
  4. Report inference regarding the comparison of women’s and men’s salaries after adjustment for degree, year of degree, starting year at UW.
  5. Report inference regarding the comparison of women’s and men’s salaries after adjustment for degree, year of degree, starting year at UW, field.
  6. Report inference regarding the comparison of women’s and men’s salaries after adjustment for degree, year of degree, starting year at UW, field, administrative duties. Save the predicted values of the mean salary for each individual as *fit5.*
  7. Report inference regarding the comparison of women’s and men’s salaries after adjustment for degree, year of degree, starting year at UW, field, administrative duties, rank.

|  |  |  |  |
| --- | --- | --- | --- |
| **Salary~** | **Ratio of Means** | **95% CI** | **P val** |
| **Male** | 1.247 | 1.208, 1.288 | .000 |
| **Male+Degree** | 1.235 | 1.196, 1.275 | .000 |
| **Male+Degree+YrDeg** | 1.113 | 1.0814, 1.147 | .000 |
| **Male+Degree+YrDeg+StartYr** | 1.116 | 1.0824, 1.1504 | .000 |
| **Male+Degree+YrDeg+StartYr+Field** | 1.0815 | 1.0505, 1.113 | .000 |
| **Male+Degree+YrDeg+StartYr+Field+Admin** | 1.0824 | 1.0524, 1.113 | .000 |
| **Male+Degree+YrDeg+StartYr+Field+Admin+Rank** | 1.0524 | 1.0276, 1.0779 | .000 |

Table of Results Using a Generalized Linear Model with Log Link.

Using a generalized linear model with a log link, we find that as we adjust for possible confounders, the log mean ratio comparing male and female salary moves towards the null hypothesis, although it fails to reach a reasonable threshold for equality. After adjusting for all variables, mean ratio salary for males is 5.24% greater than female geometric mean salary. Based on a 95% confidence interval, it would not be unusual if the true mean ratio for males were anywhere between 2.76% and 7.79% greater than for females. At the 5% significance level, we reject the null hypothesis of equal mean salary (P<0.001).

1. Briefly discuss the similarities and differences between the analyses performed in problems 3 – 5. How similar are the predicted values between the models? How different is the inference you would obtain?



1. For the analysis model that you would have chosen *a priori*, summarize the scientific relevance of the single model that you think would best reflect any discrimination against women in awarding salaries. Give a formal report of your methods and results.

The model that considers salary as a response to gender after adjusted for degree, year of degree, starting year at UW, field, and administrative duties is preferred. In particular, absolute difference in mean salary is of interest, since other measures such as ratios or percentage increase may mask the true magnitude of the sex effect on salary.

Methods: Linear regression analysis will be used to consider a difference in salary by sex. Salary in 1995 will be used as a response and sex will be used as a predictor, where sex will also be adjusted for degree, year of degree, starting year at UW, field, and administrative duties. The Huber-White Sandwich estimator will be used for estimating standard errors, confidence intervals, and 2-sided p-values.

Results: Based on a linear regression analysis, mean male salary is estimated to be $419.73 greater than mean female salary. Based on a 95% confidence interval, it would not be unusual if the true mean salary for males were anywhere between $260.47 and $578.99 greater than female salary. At the 5% significance level, we reject the null hypothesis of equal mean salaries when comparing men and women after adjusting for degree, year of degree, starting year at UW, field, and administrative duties (P<0.001).