Biost 536 / Epi 536 Categorical Data Analysis in Epidemiology

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Lecture 9:
Regression : Adjusting for Confounding

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Lecture Outline

- Regression models
  - Linear vs logistic vs Poisson vs GLM variants
- Adjustment for confounders
  - Linear continuous
  - Transformed continuous
  - Polynomial continuous
  - Dummy variables
    - Quantile categorization vs scientific categorization
  - Piecewise linear
  - Splines

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Regression Models

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Common Regression Models

- According to the parameter and contrast across groups
  - Mean (differences) → Linear regression (a GLM)
  - Mean (ratios) → GLM with log link (a GLM)
  - Geom Means (ratios) → Linear regression on logs (a GLM)
  - Odds (ratios) → Logistic regression (a GLM)
  - Rates (ratios) → Poisson regression (a GLM)
  - Hazards (ratios) → Proportional Hazards regr
  - Quantiles (ratios) → Parametric (AFT) survival regr
- A special case for this course
  - Cumulative incidence → Exponential regression (a GLM)

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General Regression

- General notation for variables and parameter

Y_i	Response measured on the *i*th subject
X_i	Value of the POI for the *i*th subject
$W_{1i}, W_{2i}, \ldots$	Value of adjustment variables for the *i*th subject
θ_i	Parameter (summary measure) of distribution of Y_i

- The parameter might be the mean, geometric mean, odds, rate, instantaneous risk of an event (hazard), etc.
- (Sometimes we will use multiple regression predictors to model the scientific POI
  - E.g., linear and quadratic terms, dummy variables, etc.)

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Multiple Regression

- General notation for multiple regression model

$$g(\theta_i) = \beta_0 + \beta_1 \times X_i + \beta_2 \times W_{1i} + \beta_3 \times W_{2i} + \cdots$$

$g(\ )$ "link" function used for modeling

β_0 "Intercept"

β_1 "Slope for Pred of Interest X"

β_j "Slope for covariate W_{j-1}"

- The link function can usually be viewed as either none (means) or log (geom mean, odds, hazard) but some exceptions
  - log odds viewed by many as logit mean
  - complementary log log link for proportions measuring cumulative incidence, which relate to log link on exponential hazards
    - log (- log *p*) = log λ + log Δt

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Generalized Linear Model (GLM)

- A special subset of my general regression model is the "Generalized Linear Model"
  - θ is the mean of response *Y*
  - *Y* has a 1-parameter exponential family distribution
    - Includes normal (Gaussian), binomial, Poisson, exponential, gamma, negative binomial, log-normal…
- Estimating equations derived from maximum likelihood theory in these "regular" models (i.e., distributions that satisfy certain technical assumptions)
  - MLEs are "consistent": arbitrarily close to the truth with large *n*
  - MLEs are "asymptotically efficient": have greatest precision possible in general (meet Cramer-Rao lower bound)

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GLM Estimating Equations

- Use maximum likelihood estimation assuming a parametric distribution for the data

$$Y_i \mid \vec{X}_i = \vec{x}_i \sim \left(\mu_i, \sigma_i^2 = V(\mu_i)\right) \qquad \vec{x}_i\vec{\beta} = \beta_0 + x_{1i}\beta_1 + x_{2i}\beta_2 + \ldots$$

"Score function estimating equation":

$$U(\beta_j) = \sum_{i=1}^{n} \frac{Y_i - \mu_i}{\sigma_i^2} \frac{\partial \mu_i}{\partial \beta} = \sum_{i=1}^{n} \frac{Y_i - \mu_i}{V(\mu_i)} \frac{\partial \mu_i}{\partial \beta} \qquad g(\mu_i) = \vec{x}_i\vec{\beta}_i$$

Find regression parameter estimates $\hat{\vec{\beta}}_i$ to satisfy :

$$U(\hat{\beta}_j) = \sum_{i=1}^{n} \frac{Y_i - \hat{\mu}_i}{V(\hat{\mu}_i)} \frac{\partial \mu_i}{\partial \beta}\bigg|_{\mu_i = \hat{\mu}_i} = 0$$

- Quasi-likelihood and generalized estimating equations (GEE) use same general equation, but do not assume parametric distribution
  - Only model mean and mean-variance relationship

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"Family": Mean Variance Relationship

- If we trust our linear model and data distribution completely, then using the correct mean-variance relationship will be more efficient
  - Weights observations less if the variance is greater
  - Works because <u>every</u> weighted average will correctly estimate the straight line relationship

Gaussian $$U(\beta)=\sum_{i=1}^{n}\frac{Y_i-\mu_i}{\sigma^2}\frac{\partial\mu_i}{\partial\beta}$$

Bernoulli $$U(\beta)=\sum_{i=1}^{n}\frac{Y_i-\mu_i}{\mu_i(1-\mu_i)}\frac{\partial\mu_i}{\partial\beta}$$

Poisson $$U(\beta)=\sum_{i=1}^{n}\frac{Y_i-\mu_i}{\mu_i}\frac{\partial\mu_i}{\partial\beta}$$

Exponential $$U(\beta)=\sum_{i=1}^{n}\frac{Y_i-\mu_i}{\mu_i^2}\frac{\partial\mu_i}{\partial\beta}$$

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Role of Link Function

- Conceptually, we can use any link function with any distribution family
- There are often advantages of using the "canonical" link for each family: These tend to be the default value in statistical software
  - Gaussian distribution ➔ Identity link
  - Binomial distribution ➔ Logit link (on means)
  - Poisson distribution ➔ Log link
  - Exponential distribution ➔ Inverse link: *g(μ) = 1/ μ*

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Binary Data: Role of Link Function

- With Bernoulli data, we might consider three different link functions

Identity $$U(\beta)=\sum_{i=1}^{n}\frac{Y_i-X_i\beta}{X_i\hat{\beta}\left(1-X_i\hat{\beta}\right)}X_i \quad\Rightarrow\quad RD$$

Log $$U(\beta)=\sum_{i=1}^{n}\frac{Y_i-e^{X_i\beta}}{\left(1-e^{X_i\hat{\beta}}\right)}X_i \quad\Rightarrow\quad RR$$

Logit $$U(\beta)=\sum_{i=1}^{n}\left(Y_i-\frac{e^{X_i\beta}}{1+e^{X_i\beta}}\right)X_i \quad\Rightarrow\quad OR$$

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Uses of Regression

- Modeling questions about associations
  - "Defining contrasts" for associations between response and POI
  - "Defining contrasts" for detecting effect modification
- Enabling comparisons across POI groups that are "otherwise similar" with respect to other variables
  - Adjusting for confounding
  - Adjusting to gain precision

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Borrowing Information

- Use other groups to make estimates in groups with sparse data
- Intuitively: 67 and 69 year olds would provide some relevant information about 68 year olds
- Assuming straight line relationship in modeled covariates tells us how to adjust data from other (even more distant) age groups
  - If we do not know about the exact functional relationship, we might want to borrow information only close to each group

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Defining "Contrasts"

- Define a comparison across groups to use when answering scientific question
- If straight line relationship in parameter, slope for POI compares parameter between groups differing by 1 unit in X when all other covariates in model are equal
- If nonlinear relationship in parameter, slope is average comparison of parameter between groups differing by 1 unit in X "holding covariates constant"
  - Statistical jargon: a "contrast" across the groups
- If multiple regression predictors model the POI, interpetation of the contrast is more difficult

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Comparison of Models

- The major difference between regression models is interpretation of the parameters
  - Summary: Mean, geometric mean, odds, hazards
  - Comparison of groups: Difference, ratio
- Issues related to inclusion of covariates remain the same
  - Address the scientific question
    - Predictor of interest; Effect modifiers
  - Address confounding
  - Increase precision

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Interpretation of Parameters

- Intercept
  - Corresponds to a population with <u>all</u> modeled covariates equal to zero
  - Most often outside range of data; quite often impossible; very rarely of interest by itself
- Slope
  - A comparison between groups differing by 1 unit in corresponding covariate, but agreeing on all other modeled covariates
    - Identity link: a difference in the summary measure
    - Log link: a ratio in the summary measure (difference in logs)
  - Sometimes impossible to use this definition when modeling interactions or complex curves
    - I most often resort to looking at predicted summary measures

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Regression with Identity Link

- E.g., modeling mean of response *Y* on predictors *X, W_1, W_2, …*

Model $\theta_i = \beta_0 + \beta_1 \times X_i + \beta_2 \times W_{1i} + \cdots$

$X_i = 0 \quad W_{ji} = 0 \qquad \theta = \beta_0$

$X_i = x \quad W_{ji} = w_{ji} \qquad \theta = \beta_0 + \beta_1 \times x + \beta_2 \times w_{1i} + \cdots$

$X_i = x+1 \quad W_{ji} = w_{ji} \qquad \theta = \beta_0 + \beta_1 \times x + \beta_1 + \beta_2 \times w_{1i} + \cdots$

Difference of θ: β_1

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Regression with Identity Link

- Most common example: Linear regression for difference in means
  - Ordinary (unweighted) least squares
    - Optimal (in some sense): independent, homoscedastic response
  - Weighted least squares: weighting by inverse variance
    - Optimal (in some sense): independent, heteroscedastic response
  - *(Biost 540) General least squares: weighting by covariance matrix*
    - *Optimal (in some sense) for correlated response data*
- Optimality: For ***any*** distribution having a variance, then
  - "Best linear unbiased estimate": greatest precision among all unbiased estimators that are a linear combination of the data
    - Gauss-Markov theorem
  - If response is normal within groups and linear model correct, then
    - Unbiased *and* efficient (greatest possible precision)
    - Also MLE

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Linear Regression: Ordinary, Weighted

- Use least squares estimation
  - Ordinary least squares: assume equal variance
  - Weighted least squares: use actual variances

$$Y_i \mid \vec{X}_i = \vec{x}_i \sim \left(\vec{x}_i\vec{\beta}, \sigma_i^2\right) \qquad \vec{x}_i\vec{\beta} = \beta_0 + x_{1i}\beta_1 + x_{2i}\beta_2 + \ldots$$

$$\text{"Least squares": Minimize} \quad \sum_{i=1}^{n} \frac{\left(Y_i - \vec{x}_i\vec{\beta}_i\right)^2}{\sigma_i^2}$$

"Score function estimating equation":

$$U\left(\beta_j\right) = \sum_{i=1}^{n} \frac{Y_i - \vec{x}_i\vec{\beta}_i}{\sigma_i^2} x_{ji}$$

Find regression parameter estimates $\hat{\vec{\beta}}_i$ to satisfy:

$$U\left(\hat{\beta}_j\right) = \sum_{i=1}^{n} \frac{Y_i - \vec{x}_i\hat{\vec{\beta}}_i}{\sigma_i^2} x_{ji} = 0$$

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Properties of Least Squares Estimates

- In unweighted regression with no covariates
  - Estimated intercept is the sample mean
- In unweighted regression with a single binary estimator X
  - Estimated intercept is the sample mean for group with X=0
  - Estimated slope is the difference in sample means
    - Group with X=1 minus group with X=0
- In unweighted regression with as many parameters as distinct combinations of predictors
  - The fitted value for every group will correspond exactly to the sample mean
- In weighted regressions the last two may not hold, unless all subjects with the same covariate values have the same weights

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Linear Regression: Binary Response

- A mean-variance relationship: mean $\mu = p$ ➔ variance $p(1-p)$
- Optimal (BLUE) estimates would use weighted least squares
  - We have to estimate the weights in an iterative search

"Score function estimating equation":

$$U(\beta_j) = \sum_{i=1}^{n} \frac{Y_i - \vec{x}_i\vec{\beta}_i}{\sigma_i^2} x_{ji} = \sum_{i=1}^{n} \frac{Y_i - \vec{x}_i\vec{\beta}_i}{\vec{x}_i\vec{\beta}_i\left(1-\vec{x}_i\vec{\beta}_i\right)} x_{ji}$$

Find regression parameter estimates $\hat{\vec{\beta}}_i$ to satisfy:

$$U(\hat{\beta}_j) = \sum_{i=1}^{n} \frac{Y_i - \vec{x}_i\hat{\vec{\beta}}_i}{\vec{x}_i\hat{\vec{\beta}}_i\left(1-\vec{x}_i\hat{\vec{\beta}}_i\right)} x_{ji} = 0$$

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Stata: Binary Response with Identity Link

- Weighted least squares using iteratively estimated weights:
  - `glm y x1 …,family(binomial) link(identity)`
  - `binreg y x1 …,rd`
  - `cs y x1`
- Ordinary (unweighted) least squares :
  - `glm y x1 …,family(gaussian) link(identity)`
  - `regress y x1 …`
- Robust standard errors can be obtained with either
  - Specify option `robust` with `glm` or `regress`
  - Specify option `vce(robust)` with `binreg`

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Examples: From Homework #2

- Modeling females' risk of dying within 4 years as a function of
  - Estrogen use (0 or 1)
  - Prior history of cardiovascular disease (0 or 1)
  - Age (65 – 100 years)
- Derived variable *estr_prev = estrogen * prevdis*
- Comparisons
  - No covariates
  - Binary covariate
  - Saturated model with two binary covariates
  - Continuous covariate

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Relative Advantages

- IF the linear model is correct, weighting by the estimated mean-variance relationship is most efficient
- HOWEVER, if the linear model is not correct, we can end up with negative variance estimates
  - Not a good thing to use when weighting
- The unweighted analysis
  - is unbiased if the linear model is correct,
  - interpretable as a linear trend when the linear model is not exactly correct, and
  - can be adjusted for the heteroscedasticity when making inference

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Regression with Log Link

- E.g., modeling geometric mean, odds, rate, hazard of response *Y* on predictors *X, W₁, W₂, …*

Model $\log(\theta_i) = \beta_0 + \beta_1 \times X_i + \beta_2 \times W_{1i} + \cdots$

$X_i = 0 \quad W_{ji} = 0 \qquad \log(\theta) = \beta_0$

$X_i = x \quad W_{ji} = w_{ji} \qquad \log(\theta) = \beta_0 + \beta_1 \times x + \quad \beta_2 \times w_{1i} + \cdots$

$X_i = x+1 \quad W_{ji} = w_{ji} \qquad \log(\theta) = \beta_0 + \beta_1 \times x + \beta_1 + \beta_2 \times w_{1i} + \cdots$

Difference of $\log(\theta)$: β_1

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Regression with Log Link: Back Transform

- E.g., modeling geometric mean, odds, rate, hazard of response *Y* on predictors *X, W₁, W₂, …*

Model $\log(\theta_i) = \beta_0 + \beta_1 \times X_i + \beta_2 \times W_{1i} + \cdots$

$X_i = 0 \quad W_{ji} = 0 \qquad \theta = e^{\beta_0}$

$X_i = x \quad W_{ji} = w_{ji} \qquad \theta = e^{\beta_0 + \beta_1 \times x + \quad \beta_2 \times w_{1i} + \cdots}$

$X_i = x+1 \quad W_{ji} = w_{ji} \qquad \theta = e^{\beta_0 + \beta_1 \times x + \beta_1 + \beta_2 \times w_{1i} + \cdots}$

Ratio of θ: e^{β_1}

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Logistic Regression

$$Y_i \mid \vec{X}_i = \vec{x}_i \sim B\left(1, \frac{e^{X_i\beta}}{1+e^{X_i\beta}}\right) \qquad \vec{x}_i\vec{\beta} = \beta_0 + x_{1i}\beta_1 + x_{2i}\beta_2 + \ldots$$

"Score function estimating equation":

Logit link $$U(\beta) = \sum_{i=1}^{n}\left(Y_i - \frac{e^{X_i\beta}}{1+e^{X_i\beta}}\right)X_i$$

Find regression parameter estimates $\hat{\vec{\beta}}_i$ to satisfy :

$$U(\hat{\beta}_j) = 0$$

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Stata: Logistic regression

- Weighted least squares using iteratively estimated weights:
  - `logistic y x1 …,` or `logit y x1 …,`
  - `glm y x1 …,family(binomial) link(logit)`
  - `binreg y x1 …,`
- Robust standard errors can be obtained with either
  - Specify option `robust` with `glm` or `logistic / logit`
  - Specify option `vce(robust)` with `binreg`

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Poisson Regression

$Y_i \mid \vec{X}_i = \vec{x}_i \sim P(\lambda_i t_i) \qquad \log(\lambda_i) = \vec{x}_i \vec{\beta} = \beta_0 + x_{1i}\beta_1 + x_{2i}\beta_2 + \ldots$

"Score function estimating equation":

Logit link $\qquad U(\beta) = \sum_{i=1}^{n} \left(Y_i - t_i e^{x_i \beta}\right) X_i$

Find regression parameter estimates $\hat{\beta}_i$ to satisfy :

$$U(\hat{\beta}_j) = 0$$

Note that we formulate regression with "exposure" having a known coefficient of 1 for $\log(t_i)$:

$$\log(\lambda_i t_i) = \log(t_i) + \beta_0 + x_{1i}\beta_1 + x_{2i}\beta_2 + \ldots$$

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Stata: Poisson regression

- Weighted least squares using iteratively estimated weights:
  - `poisson y x1 …,exposure(t)`
  - `glm y x1 …,family(poisson) link(log)`
- Robust standard errors can be obtained with either
  - Specify option `robust` with `glm` or `poisson`
  - Specify option `vce(robust)` with `binreg`

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Properties of MLEs with Canonical Link

- In regression with no covariates
  - Estimated intercept is the sample mean
- In regression with a single binary estimator X
  - Estimated intercept is the sample mean for group with X=0
  - Estimated slope is the difference in sample means
    - Group with X=1 minus group with X=0
- In regression with as many parameters as distinct combinations of predictors
  - The fitted value for every group will correspond exactly to the sample mean

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Stata: Regression Commands

- Linear regression (means, proportions)
  - `regress Y X W1 W2 W3, robust`
- Linear regression (geometric means)
  - `regress logY X W1 W2 W3, robust`
- Logistic regression (odds)
  - `logistic Y X W1 W2 W3, [robust]`
  - `logit Y X W1 W2 W3, [robust]`
- Poisson regression (rates)
  - `poisson Y X W1 W2 W3, robust`
- Proportional hazards regression (hazards)
  - `stset Y Event`
  - `stcox X W1 W2 W3, robust`

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Stata: glm Regression Commands

- Difference of means, proportions (linear regression)
  - `glm Y X W1, robust family(gaussian) link(identity)`
- Difference of log means, log proportions
  - `glm Y X W1 W2, robust link(log)`
- Difference of log geometric means (linear regression on logs)
  - `glm logY X W1 W2, robust`
- Difference of log odds (logistic regression)
  - `glm Y X W1, [robust] family(binomial) link(logit)`
- Difference of log rates (Poisson regression)
  - `glm Y X W1 W2, robust family(poisson) link(log)`
- Back transforming regression parameters with log link
  - `glm Y X W1 W2, robust family(…) link(…) eform`

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Stratification

- When we are uninterested in making inference about associations between some (categorical) variables and the response, we can consider stratified analyses
  - Perform an analysis within each stratum defined by a unique combination of the stratifying variables
  - Compute a (weighted) average of the results from those analyses
- But stratification adjusts for covariates and all interactions among those covariates
  - E.g, sex, race, and the sex-race interaction
- Our habit in regression is to just adjust for the covariates (the "main effect"), and consider interactions less often

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Combining Across Subgroups

- Based on the properties of independent, normally distributed estimates

$$\text{For independent } \hat{\theta}_1 \dot{\sim} N\left(\theta_1, se_1^2\right); \quad \hat{\theta}_2 \dot{\sim} N\left(\theta_2, se_2^2\right)$$

$$a\hat{\theta}_1 + b\hat{\theta}_2 \dot{\sim} N\left(a\theta_1 + b\theta_2, a^2 se_1^2 + b^2 se_2^2\right)$$

$$\hat{\theta}_1 - \hat{\theta}_2 \dot{\sim} N\left(\theta_1 - \theta_2, se_1^2 + se_2^2\right)$$

$$\hat{\theta}_1 / \hat{\theta}_2 \dot{\sim} N\left(\frac{\theta_1}{\theta_2}, \frac{1}{\theta_2^2}\left(se_1^2 + \frac{\theta_1^2}{\theta_2^2} se_2^2\right)\right)$$

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Stratification vs Regression

- Generally, any stratified analysis could be performed as a regression model
- Stratification adjusts for covariates and all interactions among those covariates
  - E.g, sex, race, and the sex-race interaction
- Any covariates modeled in each stratum's analysis would have to be modeled as interactions
  - E.g., sex stratified analyses of response adjusted for age could be modeled in an unstratified analysis with sex-age interaction
- Our habit in regression is to just adjust for the covariates (the "main effect"), and consider interactions less often

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Example

- We are interested in exploring the incidence of colorectal cancer by birthplace among whites in the US
  - Cases identified through the SEER registry 1973-1987
  - Available data: birthplace (US, 25 non-US, unknown), age in 5 year groups, sex
  - Denominator data from US census data
- Compare
  - Directly standardized rates using Stata `ir`
  - Poisson regression using Stata `poisson`

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Adjustment for Covariates

- We "adjust" for other covariates
- Define groups according to
  - Predictor of interest, and
  - Other covariates
- Compare the distribution of response across groups which
  - differ with respect to the Predictor of Interest, but
  - are the same with respect to the other covariates
    - "holding other variables constant"

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Adjustment for Confounders

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Adjustment for Covariates

- We "adjust" for other covariates
- Define groups according to
  - Predictor of interest, and
  - Other covariates
- Compare the distribution of response across groups which
  - differ with respect to the Predictor of Interest, but
  - are the same with respect to the other covariates
    - "holding other variables constant"

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Adjusted Analyses

- We include covariates in a regression model for three reasons
  - Modeling our primary question about associations
    - Predictor of interest
    - Effect modification
  - Adjusting for confounders
  - Adding precision to our analysis
- As a rule, it is most important to accurately model the true relationship between the response and confounders
  - We answer questions about general trends in our POI and effect modifiers
  - We tend to gain most precision (if any) from rough adjustment for precision variables

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Linear Predictors

- The most commonly used regression models use "linear predictors"
  - "Linear" refers to linear in the parameters
  - The modeled predictors can be transformations of the scientific measurements
    - Examples

$$g\left[\theta \,\middle|\, X_i, W_i\right] = \beta_0 + \beta_{\log X} \times \log\left(X_i\right)$$
$$g\left[\theta \,\middle|\, X_i, W_i\right] = \beta_0 + \beta_X \times X_i + \beta_{X^2} \times X_i^2$$

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Transformations of Predictors

- We transform predictors to provide more flexible description of complex associations between the response and some scientific measure
  - Threshold effects
  - Exponentially increasing effects
  - U-shaped functions
  - S-shaped functions
  - etc.

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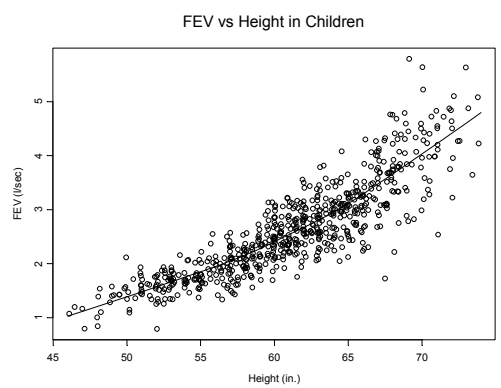
Ex: Cubic Relationship

- Linear regression



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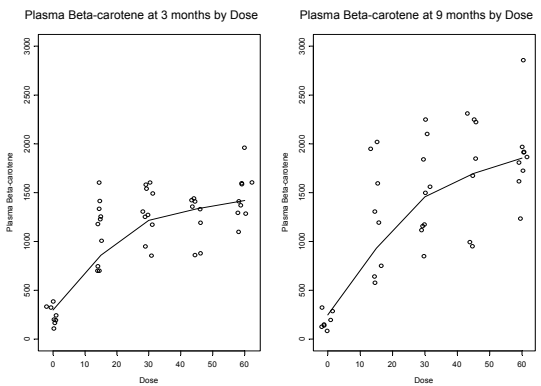
Ex: Threshold Effect of Dose?

- Linear regression



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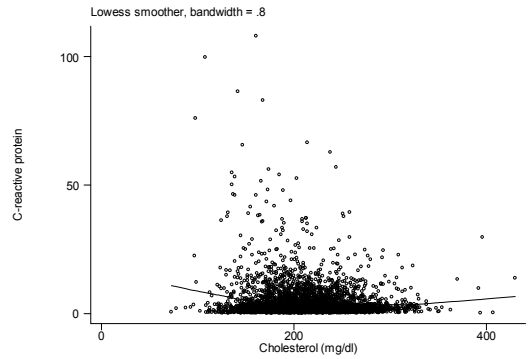
Ex: U-shaped Trend?

- Inflammatory marker vs cholesterol
  - Linear regression



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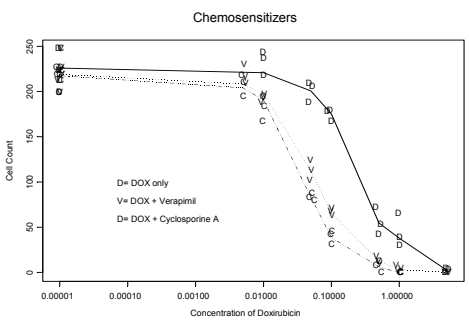
Ex: S-shaped trend

- *In vitro* cytotoxic effect of Doxorubicin with chemosensitizers
  - Poisson regression



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"1:1 Transformations"

- Sometimes we transform 1 scientific measurement into 1 modeled predictor
  - Ex: log transformation will sometimes address apparent "threshold effects"
  - Ex: cubing height produces more linear association with FEV

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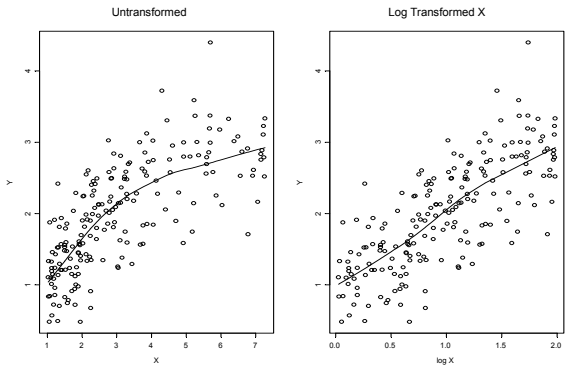
Log Transformations

- Linear regression



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"1:Many Transformations"

- Sometimes we transform 1 scientific measurement into several modeled predictor
  - Ex: "polynomial regression"
  - Ex: "dummy variables" ("factored variables")
  - Ex: "piecewise linear"
  - Ex: "splines"

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Polynomial Regression

- Fit linear term plus higher order terms (squared, cubic, …)
- Can fit arbitrarily complex functions
  - An n-th order polynomial can fit n+1 points exactly
- Generally very difficult to interpret parameters
  - I usually graph function when I want an interpretation
  - Use Stata's `predict` command
- Special uses
  - 2nd order (quadratic) model to look for U-shaped trend
  - Test for linearity by testing that all higher order terms have parameters equal to zero

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Stata: "Predicted Values"

- After computing a regression model, Stata will provide "predicted values" for each case
  - Covariates times regression parameter estimates for each case
  - ***"predict varname"***

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Dummy Variables

- Indicator variables for all but one group
  - This is the only appropriate way to model nominal (unordered) variables
    - E.g., for marital status
      - Indicator variables for
        - married (married = 1, everything else = 0)
        - widowed (widowed = 1, everything else = 0)
        - divorced (divorced = 1, everything else = 0)
        - (single would then be the intercept)
  - Often used for other settings as well
  - Equivalent to "Analysis of Variance (ANOVA)"

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Ex: Interpretation of Slopes

- Based on coding used
  - Intercept corresponds to "reference" group
  - Slope for other terms compare each group to the "reference"
- Tests for association must test all predictors together
- Need to be very careful in overinterpreting "statistical significance" among groups
  - (More later)

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Stata: Dummy Variables

- Stata has a facility to automatically create dummy variables
  - Old way: Prefix regression commands with "`xi: …`"
  - Now just prefix variables to be modeled as dummy variables with "`i.varname`"
  - (Stata will drop the lowest category by default)

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Continuous Variables

- We can also use dummy variables to represent continuous variables
  - Continuous variables measured at discrete levels
    - E.g., dose in an interventional experiment
  - Continuous variables divided into categories

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Relative Advantages

- Dummy variables fits groups exactly
  - If no other predictors in the model, parameter estimates correspond exactly with descriptive statistics
- With continuous variables, dummy variables assume a "step function" is true
- Modeling with dummy variables ignores order of predictor of interest

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Flexible Methods

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Flexible Modeling of Predictors

- We do have methods that can fit a wide variety of curve shapes
  - Dummy variables
    - A step function with tiny steps
  - Polynomials
    - If high degree: allows many patterns of curvature
  - Piecewise linear or piecewise polynomial
  - Splines
    - Piecewise linear or piecewise polynomial joined at knots
  - Fractional polynomial

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Stata: Linear Splines

- Stata will make variable that will fit piecewise linear curves
  - Joined at "knots"
  - Lines in between
- **`mkspline` *`newvar0 #k1 newvar1 #k2 newvar2 … #kp varp= oldvar`***
  - Regression on *newvar0 … newvarp*
    - Straight lines between min and k1; k1 and k2, etc.

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Stata: fracpoly

- Stata will make variables modeling "fractional polynomials"
  - Can fit many different shapes depending on degree of the fractional polynomials
  - Can ask Stata to find "best" degree of the fractional polynomials: `"fracpoly"`
  - Can ask Stata to make new variables to model fractional polynomial of desired degree: `"fracgen"`

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fracpoly

- Command

`fracpoly` *`regressioncommand yvar xvar,`* `degree(`*`#`*`)`

Example

`fracpoly regression logslry yrdeg, degree(`*`3`*`)`

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