

**Supplemental written problems due Friday, January 30, 2004 at the beginning of class.**

1. Let  $Y_i, i = 1, \dots, n$  be independent exponential random variables with  $Y_i \sim \mathcal{E}(\log(2)/\theta)$  (so  $F_Y(y) = 1 - \exp(-\log(2)y/\theta)$  for  $0 < y < \infty$ ).
  - a. Find the parametric MLE of the median of the distribution of  $Y_i$ . Derive its asymptotic distribution.
  - b. Find the asymptotic distribution of the sample median.
  - c. What is the asymptotic relative efficiency of the two estimators found in parts 1a and 1b?
  - d. Now suppose that the true distribution of the independent  $Y_i$ 's is as lognormal  $Y_i \sim LN(\mu, \sigma^2)$ , having density

$$f_Y(y) = \frac{1}{\sqrt{2\pi}\sigma y} \exp\left(-\frac{(\log(y) - \mu)^2}{2\sigma^2}\right) \mathbf{1}_{[y>0]}.$$

Further suppose  $\mu = \log(\theta)$ . For what function of  $\theta$  is the estimator you found in part a consistent? What is the asymptotic distribution of the estimator from part 1a under this new distribution for  $Y$ ?

2. Let  $Y_i, i = 1, \dots, n$  be independent lognormal random variables with  $Y_i \sim LN(\log(\theta), \sigma^2)$ .
  - a. Find the parametric MLE of the median of the distribution of  $Y_i$ . Derive its asymptotic distribution.
  - b. Find the asymptotic distribution of the sample median.
  - c. What is the asymptotic relative efficiency of the two estimators found in parts 2a and 2b?
  - d. Now suppose that the true distribution of the independent  $Y_i$ 's is as exponential  $Y_i \sim \mathcal{E}(\log(2)/\theta)$  as in problem 1. For what function of  $\theta$  is the estimator you found in part 2a consistent? What is the asymptotic distribution of the estimator from part 2a under this new distribution for  $Y$ ?
3. Discuss the relative merits of using parametric versus nonparametric estimators relative to your results in problems 1 and 2.