Stat 513
Emerson, Win 16
Homework #3
January 25, 2016

Written solutions to the homework problems are due on Wednesday, February 3, 2016 at the beginning of class.

The homework problems are divided into "regular" and "more involved" problems. In order to facilitate multiple graders, you should hand in these categories of problems separately. That is, hand in one paper that contains only the "regular" problems, and another paper that contains only the "more involved" problems (there are none for this homework).
As noted on the syllabus, copying of homework solutions is not allowed and, when detected, will be investigated as an infraction of the academy integrity policy of the University of Washington. While it is permissible to discuss problems with other students, TAs, or the instructor in order to learn how to solve a problem, your written solutions must be prepared without directly referencing any notes or solutions derived from other students or sources found on the internet.

REGULAR PROBLEMS

1. Consider an exponential regression models relating response $\vec{Y}$ to an intercept and one predictor vectors $\vec{X}$. Suppose that the elements of $\vec{Y}$ are totally independent with $Y_i \sim \mathcal{E}(\lambda_i)$ with density
$$f_{Y_i}(y) = \lambda_i e^{-\lambda_i y} \mathbf{1}_{(0,\infty)}(y),$$
where we have regression model
$$\log(\lambda_i) = \beta_0 + x_i \beta_1.$$
Derive expressions for the efficient score vector $\vec{\mathcal{U}}(\vec{\beta})$ and the Fisher's information matrix $\mathbf{J}(\vec{\beta})$.

2. Suppose $Y_1, Y_2, \ldots, Y_n$ are i.i.d. Weibull random variables having cumulative distribution function
$$F_{Y_i}(y) = \left(1 - \exp\left\{-(\lambda y)^p\right\}\right) \mathbf{1}_{(0,\infty)}(y)$$
for $\lambda \in (0,\infty)$, $p \in (0,\infty)$. Denote the unknown parameter vector by $\vec{\theta} = (p, \lambda)$ and derive expressions for the efficient score vector $\vec{\mathcal{U}}(\vec{\theta})$ and the Fisher's information matrix $\mathbf{J}(\vec{\theta})$.